

## El uso de mapas conceptuales para la expresión de algoritmos recursivos

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**Resumen** Estudiamos el uso de mapas conceptuales en la escritura de algoritmos recursivos. Los usamos para contestar preguntas del tipo *cuanto* es el valor de una función. Los mapas conceptuales son herramientas graficas que utilizamos para organizar y representar conocimiento. Incluyen conceptos y relaciones entre conceptos. Los conceptos se escriben dentro de cajas y las relaciones son palabras que se escriben sobre lineas que conectan conceptos. Usamos coincidencia de patrones (pattern matching) para separar los casos de argumentos de funciones o condiciones sobre los argumentos. Tenemos conceptos para cada posible valor de los argumentos. La recursión es explicita. Le damos un nombre al resultado de cada llamada recursiva (en caso de funciones) e indicamos el valor de los parametros de la función. Tenemos también procedimientos recursivos en cuyo caso usamos la llamada recursiva. Es una alternativa a los diagramas de flujo donde en vez de rombos tenemos recursión.

**Keywords:** Didactica · Mapas conceptuales · Programación · Recursión · Coincidencia de Patrones.

# The use of conceptual maps for the expression of recursive algorithms

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**Abstract.** We study the use of conceptual maps to write recursive algorithms. They are written to answer questions of the kind how much is the value of a function. Conceptual maps are graphic tools to organize and represent knowledge. They include concepts and relationships between concepts. Concepts are written inside boxes and the relationships are words written over a line that connects concepts. We use pattern matching to separate the cases of function arguments or conditions over the arguments. We have concepts for each possible value of the arguments. Recursion is explicit. We give a name to the result of each recursive call (in case of function) and indicate the value of the parameters to the function. We have also recursive procedures in which case we use the recursive call. It is an alternative to flowcharts where instead of diamonds we have recursion.

**Keywords:** Didactics · Conceptual maps · Programming · Recursion · Pattern Matching

## 1 Introduction

Conceptual maps are graphic tools to organize and represent knowledge. They include concepts and relationships between concepts. Concepts are written inside boxes and the relationships are words written over a line that connects concepts.

There are propositions. Propositions contain two or more concepts connected by linking phrases that form a meaningful affirmation. The relationships usually consist of verbs forming propositions for each pair of concepts. By convention, links run top-down unless annotated by an arrow. More general concepts are above less general concepts.

Concepts are usually nouns and linking phrases or relationships are usually verbs. In well constructed concept maps, concepts and links are as short as possible, possibly single words and the structure is hierarchical where the root of the map is a representative of the topic of the map. We study the use of conceptual maps to write recursive algorithms. They are written to answer questions of the kind how much is the value of a function. We use pattern matching to separate the cases of function arguments or conditions over the arguments. We have concepts for each possible value of the arguments. Recursion is explicit. We give a name to the result of each recursive call and indicate the value of the parameters

to the function. By example we write below the map for the factorial function. We have also recursive procedures in which case we use the recursive call.

In section 2 we present Conditions and Pattern Matching. In section 3 we present Concept Maps. In section 4 Operations on datatypes. In section 5 the table of truth function, in section 6 further work and in section 7 conclusions.

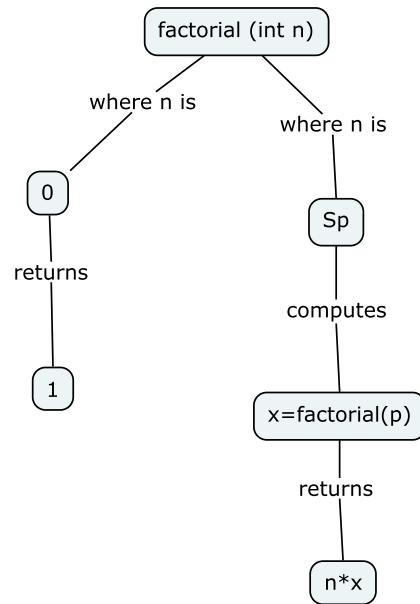


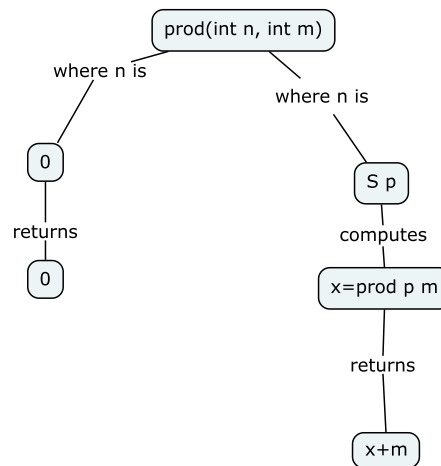
Fig. 1. Factorial function

## 2 Conditions and Pattern Matching

The definition of a function by pattern matching is common and quite important in practice. Given an inductive definition of a set, a definition by pattern matching over the set defines the value of the function for each constructor of the set. By example the factorial function defined above gives values for 0 and for (S p) where S is successor. There are two requirements for the correctness of a pattern matching definition. The first is that recursive definitions are well-founded (computed in smaller arguments). The second is that the equations cover all possible cases of the arguments. This is ensured by imposing definitions to be exhaustive and mutually disjoint (T.Coquand, 1992). Other examples of pattern matching definitions presented afterwards are the operations on lists and on binary search trees. Besides pattern matching definitions we have other kinds of exhaustive and mutually disjoint definitions, by example: two integers are equal

or different, or trichotomy: given two integers  $a$  and  $b$  holds:  $b==a$ ,  $b<a$  or  $b>a$ . This kind of definitions are used below in the operations on lists and on trees. Another example is a function that computes the product between two natural numbers. It can be defined as above.

Recursive calls are concepts. Conditions that express pattern matching are also concepts. We don't have diamonds as in flowcharts, instead we have recursion. We have concepts for each possible value of the arguments or each possible condition. We have relationships of the kind "computes", "realize", "when", "where" and "returns". The results of recursive calls are concepts with a link that gets in called computes. The result of the function is returned in the concepts at the leaves. The name and argument of the functions are given by the concept at the root. Software tools like CmapTools can be used to draw these maps. This facilitates the collaborative work in groups.



**Fig. 2.** The product function

### 3 Concept Maps

Concept Map is a graphical tool that enables people to express knowledge in a form easily understood by others. Represent meaningful relationships between concepts. Consist in concepts represented in boxes connected by arcs that relate pair of concepts (Reiska et al, 2016; Schroeder et al, 2018). In our case, the concepts represent values and the relationships usually consist of verbs that relate values of the kind "n is", "computes" and "returns". The relationships are transitive and say how to compute values.

Concept maps constructed by different persons can be different. There can be many different representations of the same function that are correct. In a concept

map, the root node, represents the topic of the map. It is an effective way of representing and communicating knowledge. From an educational perspective, the use of concept maps can facilitate teaching and learning (Cañas et al, 2005).

Usually in concept maps, there are no restrictions on the words we can use to form concepts and linking phrases. Concepts are usually nouns and linking phrases are usually verbs and is recommended that they consist of as few words as possible.

One way to organize knowledge hierarchically in a rather simple and compact way are concept maps (Cañas 2015).

Each path from the root to a leave can be read as an individual statement that makes sense. Concepts and linking phrases are as short as possible and the structure is hierarchical being the root a representative of the topic of the map. We regard concept maps as the visualization of knowledge (Cañas et al, 2005).

Since the 70's, Concept Maps has been used to represent knowledge in different domains. Software packages like CmapTools facilitate online manipulation of concept maps and knowledge sharing, organization and browsing. Concept maps give a representation of an expert domain knowledge in a easily understood by others form (Carvalho et al, 2001).

A concept map is a two-dimensional representation of a set of concepts constructed in a way such that the interrelationship between them is evident. Represent meaningful relationships between concepts in the form of propositions. These are concepts linked by words that form a semantic unit. In our case, propositions are paths formed by concepts and relationships, from the root to the leaves (Carvalho et al, 2001).

### **3.1 Searching the Web from a Concept Map**

Concept maps have a hierarchical structure, with more general concepts at the top and more specific at the bottom. Other information as the number of ingoing and outgoing links of a concept provide additional information. A concept map represents a person understanding of a particular domain of knowledge. Concepts and relationships are interpreted within that domain. CmapTools allows to search the web from a concept map. The user constructs a concept map and CmapTools takes it as the basis for the search (Cañas et al, 2005).

The Web-search algorithm allows the user to select a concept and asks the system to search information relevant to the concept. The information from the web aids the user in constructing knowledge representation.

### **3.2 Concept Maps as graphical Representation of Knowledge Models**

The concept maps are graphical representations of segments of information or conceptual knowledge and usually have similarities and differences. A concept map is a hierarchical structure by different levels of generality or conceptual inclusivity, formed by concepts, propositions and linking words (Rodriguez Cruz, 2007).

Concept maps can be used as strategy before, during or after the instruction, because allow the incorporation of new concepts before deepen in them, their group construction, during the process of teaching-learning and a revision that the concepts were finally understood (Rodriguez Cruz, 2007).

Is necessary, before employing the conceptual maps, to know how to elaborate them and indicate it to the students, preferably doing examples with them.

Steps to define concept maps (Rodriguez Cruz, 2007).:

1) make a list of the involved functions 2) clasify by levels of abstraction or inclusivity establishing relationships of super, co or subordination existing between the values. 3) identify the more important function. If is of greater level of inclusivity than the others locate it in the top of the map. 4) from the clasification made in 2) construct a first conceptual map. Remember the map must be organized hierarchically by levels of inclusivity and that the values must be linked between them. 5) consider the possibility of using crossed links. 6) reelaborate the map. This allows to identify new relations between the implied concepts. 7) If will be presented to the students, acompany the presentation with an explanation.

Benefits that offer (Rodriguez Cruz, 2007).:

1) the graphical representation between the functions and the relationship between them. 2) facilitates the exposition and explication of concepts. 3) negotiation of meaning between students and teacher. 4) revision of themes and articulation between them. 5) activate previous knowledge and/or determine the level of comprehension of the functions.

Recomendations (Rodriguez Cruz, 2007).:

1) Make sure the students comprehend the basic sense of concept maps. 2) A concept map is enriched if is acompained of explanations and comments that deepen the concepts.

Concept maps are an important evaluation tool when they have been used for teaching and learning. So important as the building of concept maps is their assessment. Only when conceptual maps are used to facilitate learning can be used for evaluation and helps to learn (Marriot & Lupion Torres, 2016).

### 3.3 Buiding in Cmap Tools to Construct a Model for Education

There is agreement today that every learner must construct her knowledge through her own effort. Concepts maps have been used in education and training. They have been shown to be an effective tool for evaluation, summarizing what has been learned, planning, consolidating educational experiences, improving conditions for learning, supporting cooperation and collaboration and organizing content. Good theory-based use of the appropriate technology can increase the benefits of using concept maps in education. (Novak & Cañas, 2004).

CmapTools were designed with the objective of supporting collaboration and sharing. A collection of CmapServers where any Internet user can create a folder and construct, copy or publish their concept maps facilitates sharing and collaboration during concept map construction. A CmapServer can be installed in

a classroom to facilitate collaboration. If many users try to edit the same concept map at the same time, the program will establish synchronous collaboration where the users can modify the map and communicate via chat. When a user creates a folder in a public place, becomes the administrator and can determine which users receive permission to comment, to write or read the map (Novak & Cañas, 2004).

Novak and Gowin (1984) have represented the act of mapping as a creative activity, in which the learner must clarify meanings, identify important concepts, relationships, and structure within a specific domain of knowledge. Concept maps facilitate knowledge creation in a discipline.

Our plans are to develop concept maps in some areas of science, particularly programming, mathematics and logic.

### 3.4 A Theory of education: Meaningful Learning

Novak's theory of education includes five elements: teacher, learner, subject matter, context and evaluation which must be integrated constructively to obtain meaningful learning. Concept Mapping is used to get meaningful learning and a new model for education is presented. (Novak, 2011).

As is said by Novak, Ausubel makes a sharp distinction between learning by rote where the learner makes no effort to incorporate new concepts and meaningful learning where the learner integrates new knowledge to existing one.

As has said Novak in (Novak, 2011):

“Human beings think, feel, and act. Every learning event involves to a greater or lesser degree all three of these actions. In rote learning, there is often little emotional commitment other than to recall the information, and the extrinsic motivation that comes with getting the right answer. In meaningful learning the recognition of how the new information integrates with prior knowledge and “makes sense” provides much more rewarding intrinsic motivation. Moreover, when the learning is integral to some activity and helps to guide and clarify the activity, there is usually a higher level of positive affect resulting.”

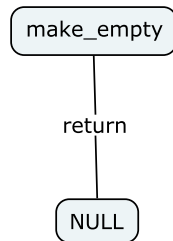
## 4 Operations in data types

In this section we study the application of conceptual maps to define recursive functions and procedures over lists and trees.

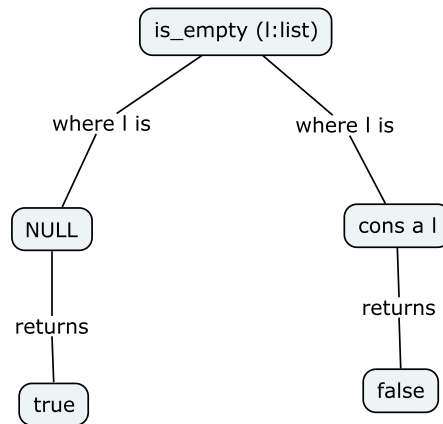
### 4.1 Operations on lists

We use lists with constructors `NULL` and `cons`. We present the following operations on lists: `make_empty_list` (Fig. 3) that constructs an empty list, `is_empty` (Fig. 4) that returns `true` if the list is empty `false` in other case, `belongs` (Fig. 5) that given a list and an integer returns a truth value, `true` if the integer belongs to the list, `false` if does not belong, `insert` (Fig. 6) that inserts an element in numerical order if it does not belong to the list, `delete` (Fig. 7) that erases an

integer if belongs to the list and print (Fig. 8) that prints the elements in the list from left to right.



**Fig. 3.** The make\_empty\_list function



**Fig. 4.** The is\_empty function



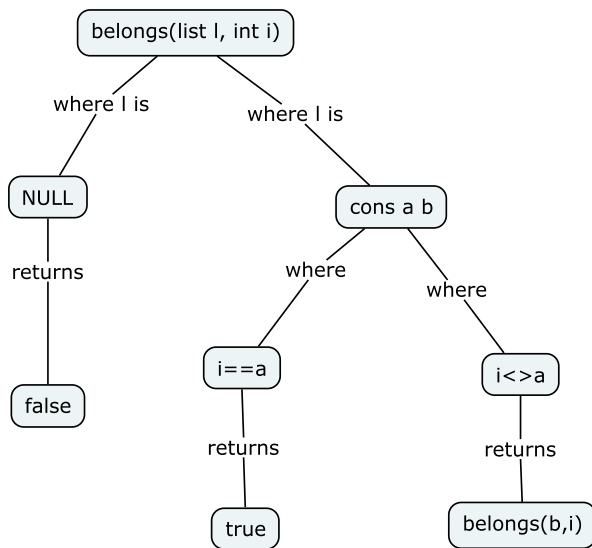


Fig. 5. The belongs function

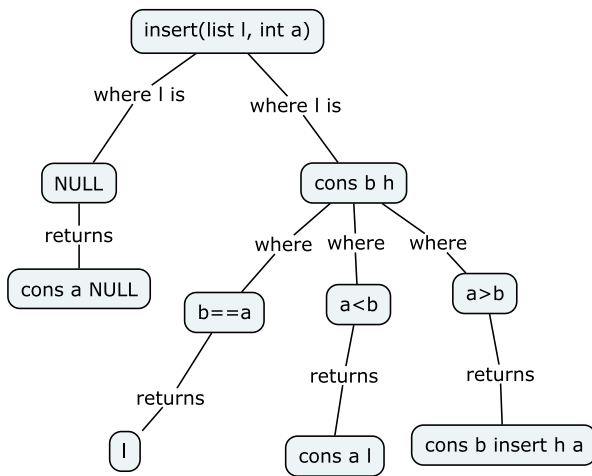


Fig. 6. The insert function

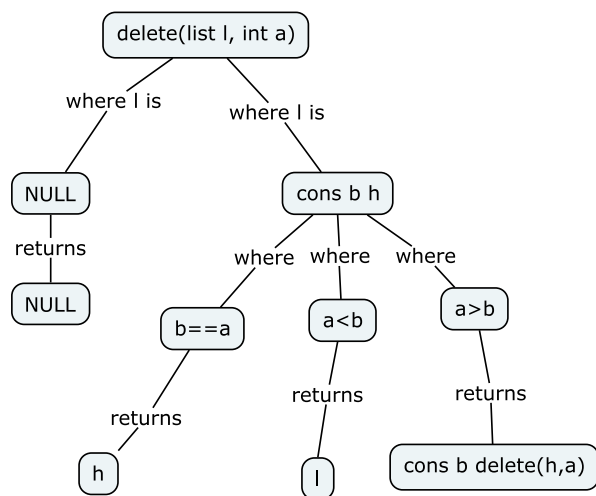


Fig. 7. The delete function

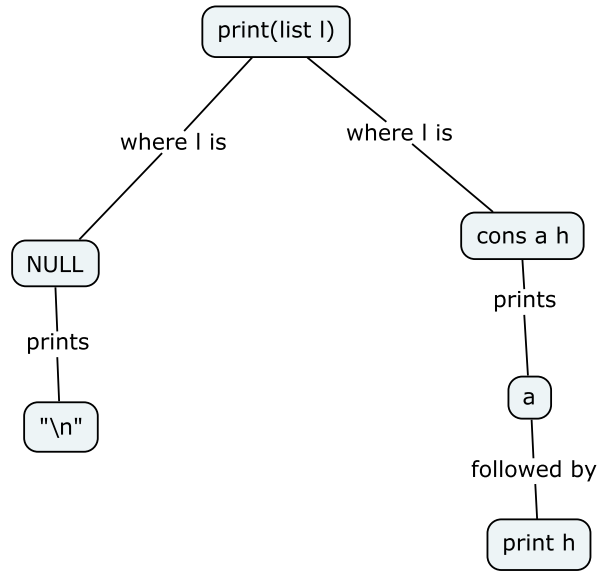
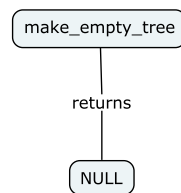


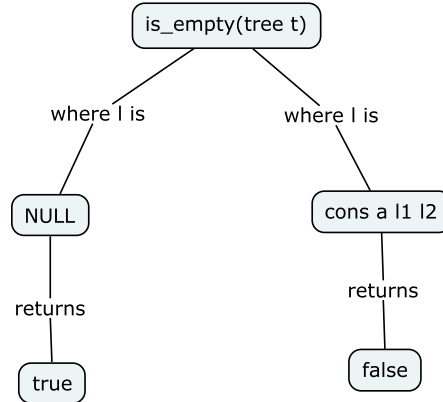
Fig. 8. The print list function

## 4.2 Operations on binary search trees

We present operations `make_empty` (Fig. 9) that constructs an empty tree, `is_empty` (Fig. 10) that returns the truth value `true` if a tree is empty, `false` otherwise, `belongs` (Fig. 11) that returns a boolean indicating if an integer belongs to a binary search tree, `insert` (Fig. 12) that inserts an integer to a binary search tree if it does not belong to it, `delete` (Fig.13) that removes an integer from a tree (if it belongs) and `inorder` (Fig. 14) that prints the elements of the tree in this order. As constructors for binary search tree we use `NULL` and `cons`. `cons` is applied to an integer and two subtrees.



**Fig. 9.** The `make_empty_tree` function



**Fig. 10.** The `is_empty_tree` function

## 5 The table of truth function

We define a conceptual map to compute a table of truth function with connectives disjunction, conjunction and negation (Fig. 16). Since this connectives form a

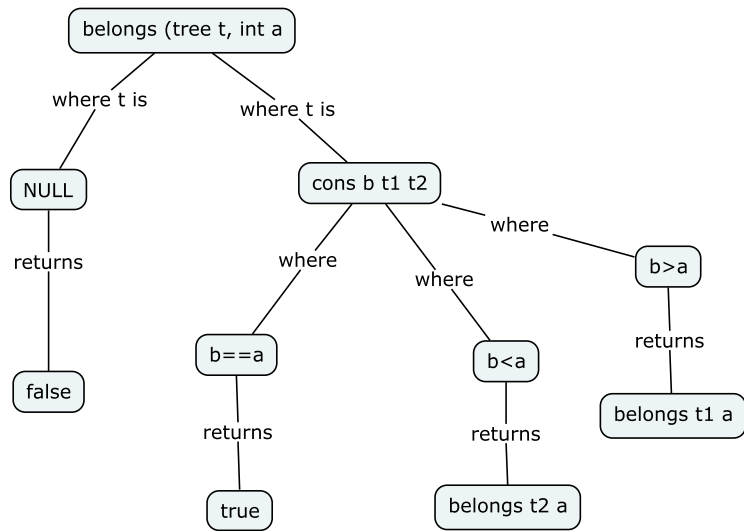


Fig. 11. The belongs to a tree function

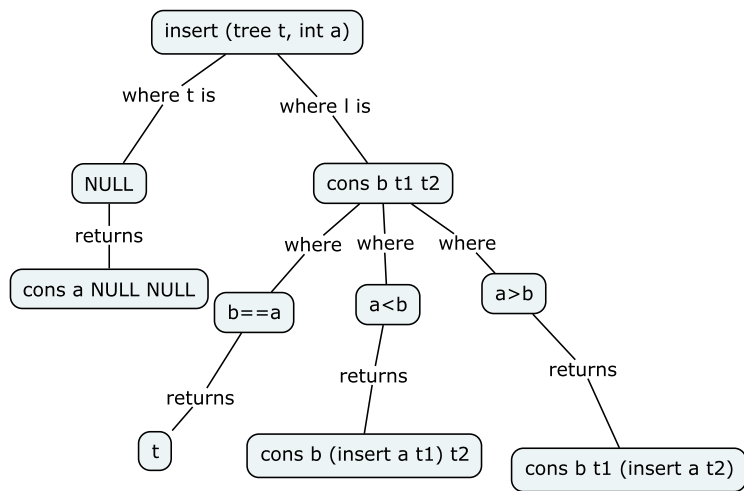


Fig. 12. The insert in a tree function

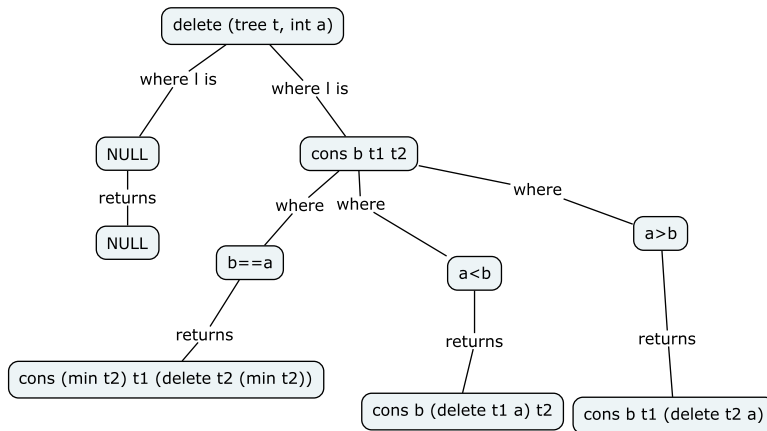


Fig. 13. The delete in a tree function

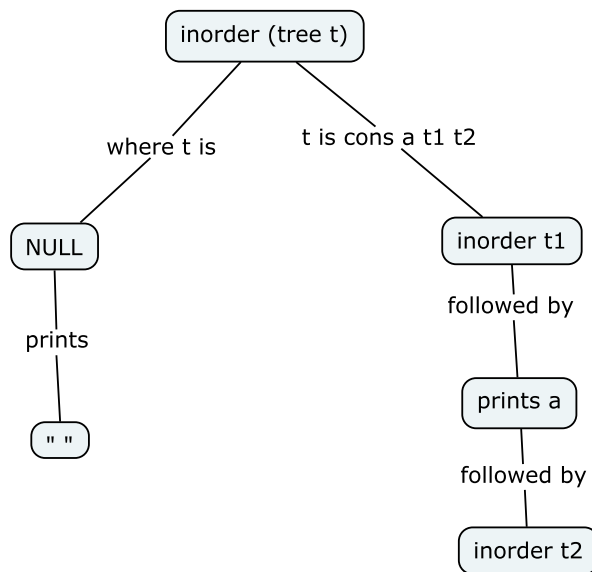


Fig. 14. The print in inorder function

complete set of connectives we don't need to include implication ( $A \rightarrow B$ ) which can be represented by  $(\neg A \vee B)$ . When we have a non atomic formula with a connective C, we compute their subformulas to atomic form and apply C to the result.

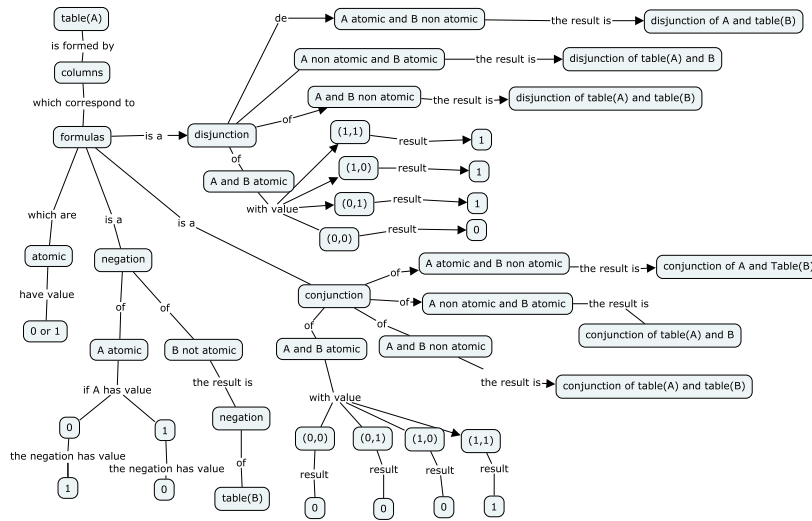


Fig. 15. A table of truth function

## 6 Further Work

To study scoring methods and metrics that are often used to describe maps like number of concepts, number of propositions, number of links, etc. (Reiska, P. et al, 2016). In our case we can use as measure the number of recursive calls, number of returned values between others. We plan to use concept maps for assessment because it illustrates different aspects of student's knowledge and can be used as a measure of learning (Reiska, P. et al, 2016). To examine how the structure (saliency of the spatial relationships between individual concepts) and the complexity (number of nodes per concept) influence learning (Krieglstein, F. et al, 2022). Concept maps were found to have positive effects on learning. However empirically documented recommendations on how to design optimal concept maps require more examinations (Krieglstein, F. et al, 2022).

## 7 Conclusions

We have studied the use of conceptual maps to express recursive algorithms in programming languages. It can be applied in structured languages and in object

oriented languages. Is a kind of flowchart with recursion instead of iteration. We haven't experimented with the tool yet. We pretend experiment with it in the next programming course.

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